# Concepts in Condensed Matter Question sheet 2

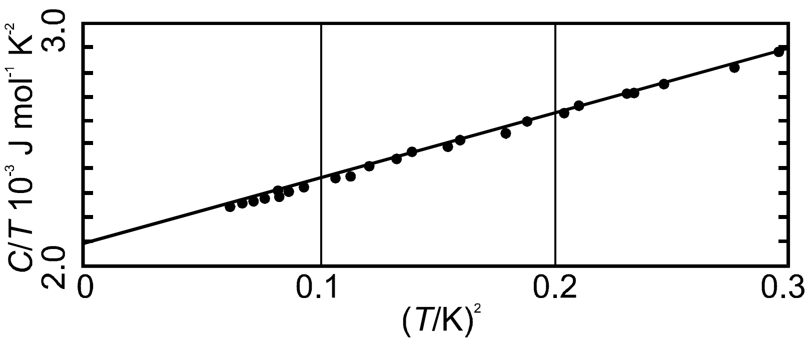
1 In lectures it was shown that the density of states in a free electron gas in three dimensions was proportional to the square root of the electron energy, i.e. $g(\varepsilon) \propto \varepsilon^{1/2}$. Deduce the dependence of $g(\varepsilon)$ on the energy for a 1 and 2 dimensional electron gas – *i.e.* a gas of electrons confined to move along a line or within a plane, and derive expressions for the Fermi energy for these two cases given a number density of n per unit length/unit area.

2 In a system of non-interacting fermions, the probability that a state is occupied is given by the function $p_f(\varepsilon) = 1 / [\ exp\{(\varepsilon - \mu(T)\ ) / kT\} + 1]$.

For two energies $\varepsilon_1$ and $\varepsilon_2$, where $\varepsilon_2 - \mu = \mu - \varepsilon_1$, show that $p_f(\varepsilon_1) = 1 - p_f(\varepsilon_2)$. Use the result to demonstrate that the dependence of $\mu$ on $T$ can be ignored, if the density of states $g(\varepsilon)$ is sufficiently slowly varying over an energy range of order $kT$ on either side of $\varepsilon_f = \mu(0)$.

Confirm that this is likely still to be a good assumption even at room temperature for the valence electrons in a typical metal where $\varepsilon_f = 5$eV.

3 What are the two main contributions to the thermal capacity of a metal, and what is the temperature dependence of each contribution in the low-temperature limit? Explain why the low-temperature thermal capacity for a metal is often plotted in the form $C/T$ against $T^2$. The graph below shows such data for potassium below approximately 0.5K. Deduce the density of states at the Fermi level, $g(E_F)$, and the Debye Temerautre, $\theta_D$ for potassium. Compare these values respectively with the value of $g(E_F)$ calculated in the free electron approximation and with the accepted values of $\theta_D = 100$K. ( For K, atomic weight = 39, density = 860 Kg m$^{-3}$)




4 Show that the mean energy of an electron in a metal (at $T$=0K) is $\overline{E} = 3E_F/5$. Explain why the Fermi level increases when a metal is compressed. Show the Bulk modulus of conduction electrons, with density $n$, is

$$B = -V(\partial P/\partial V)_T = 2nE_F/3$$

Calculate the value for copper and compare the result with: (a) the experimental value of 1.4 x $10^{11}$ N m$^{-2}$.; (b) the bulk modulus of a perfect gas at STP (N.B. in an adiabatic change d$U$ = -$p$d$V$. For Cu, atomic weight = 63.5; density = 8900 Kg m$^{-3}$.)

5 The concept of a Fermi energy can be applied much wider than just electrons in solids. A good example is that of $He^3$, which is a Fermion of spin ½, and near absolute zero is a liquid of density $81 kgm^{-3}$. Calculate the Fermi energy of the $He^3$ and comment on why it is so much smaller than a typical Fermi energy of an electron in a metal such as sodium or copper.

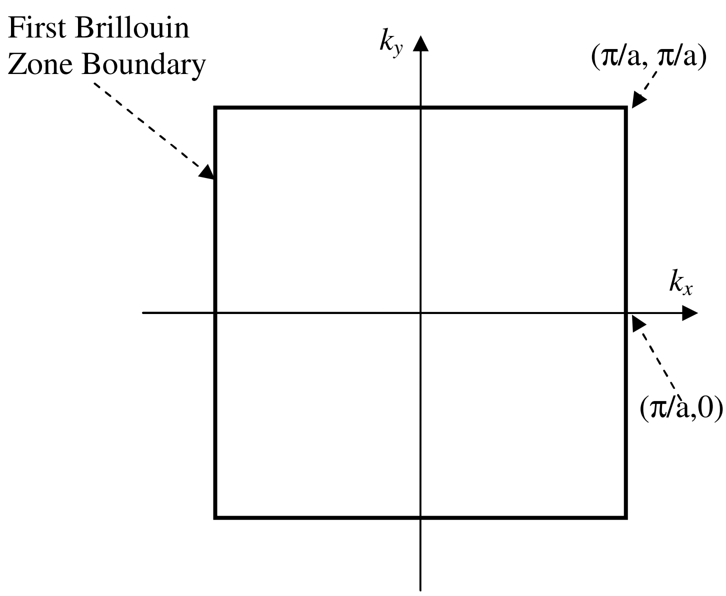
6 Why does the thermal conductivity of copper drop only slightly between 100ºC and 500ºC (values 394 and 367W/mK respectively) whereas the thermal conductivity of a stainless steel (304 grade) increases significantly over the same temperature range (values 16.2 and 21.5W/mK respectively).

7 Explain why a good approximation to the lowest energy 1D states for a chain of atoms in the nearly free electron model with crystal momentum $\hbar q$ can be obtained by expressing the state as the sum of just two plane wave states with wave-vectors $q$ and $q$-$G_0$, where $0 < q < \pi/a$ and $G_0 = 2\pi/a$. If the electrons feel a potential of $V(x) = \sum_n V_n \cos(2n\pi/a)$ where $a$ is the lattice constant of the chain, show that the Schrödinger equation may be written in matrix form as:

$$\begin{pmatrix} E_q & V_1/2 \\ V_1/2 & E_{q-G} \end{pmatrix} \begin{pmatrix} C_q \\ C_{q-G} \end{pmatrix} = \varepsilon_q \begin{pmatrix} C_q \\ C_{q-G} \end{pmatrix}$$

where $C_q$ and $C_{q-G}$ are the coefficients of the two plane wave states respectively. Give expressions for $E_q$ and $E_{q-G}$. Find an expression for the ratio of the coefficients $C_{q-G}/C_q$ and sketch its value for $0 < q < \pi/a$, explaining what the q dependence of the ratio means in terms of the nature of the q states that solve the Schrodinger equation.

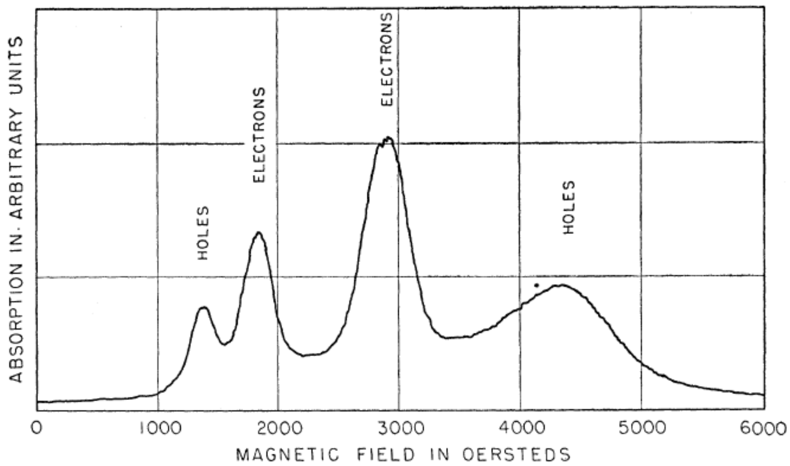
8 Explain how the size of the energy gap along the first Brillouin zone boundary of a two dimensional divalent material controls whether the material is a conductor or an insulator, and give a condition in terms of the energies either side of the band gap at the two points $(\pi/a, 0)$ and $(\pi/a, \pi/a)$ for the material to be an insulator. (The material is assumed to have a simple square unit cell with one atom per unit cell, and a unit cell side length of $a$)



9 What is meant by conduction arising from the movement of 'holes', and why is it useful in describing the conduction or certain materials? What is a 'hole' band?

Show that in a magnetic field of strength B, current carriers perform 'cyclotron' orbits with frequency $\nu = \frac{eB}{2\pi m^*}$ where m* is the effective mass of the carrier and $e$ is its charge.

The figure shows the adsorption by a silicon sample of microwaves of frequency 24GHz. Deduce the effective mass of the different types of holes and electrons. Why might one see more than one value for the effective mass of a particular sort of carrier? (Note 1T is $10^4$ Oersted.)




10 Indium antimonide has a dielectric constant ε=17 and an electron effective mass at the bottom of the conduction band of $m^*=0.014m_e$. Explain why donors are expected to produce states just below the bottom of the conduction band and calculate:
(a) how far in energy below the conduction band the lowest donor level will be,
(b) the radius of the ground state orbit,
(c) the donor concentration $n_{\text{critical}}$ at which orbits around adjacent impurities begin to overlap.
Explain why at low temperatures the donors do not contribute to conductivity unless the donor concentration is above $n_{\text{critical}}$ .

11 Draw diagrams showing how the chemical potential and carried concentrations vary across a *pn* junction. Explain why the junction shows passes very different currents for forward and reverse biasing, giving a dependence of current on biasing voltage of:

$$I = I_0\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$

What is meant by Zener breakdown. What will control the voltage at which it occurs?
Explain how a *pn* junction can be configured and made to operate as a light emitting diode, a laser and a solar cell.