IB Advanced Physics **Statistical Physics Examples 2** W Allison

Lent and Easter Term 2005

1 The CO molecule has a vibrational frequency, ω, of 4 x 10^{14} rad s^{-1}. Estimate the vibrational contribution to the heat capacity of the gas at 300K.

2 The energy density of radiation from the sun has a maximum at a wavelength of 495 nm. Assuming the sun radiates as a black body, obtain an approximate value for the brightness temperature.

3 A rubber band stretched to a constant length has a tension, f, which is proportional to T. Show that the internal energy depends only on T and that the entropy decreases when it is stretched isothermally. (a) What happens to the temperature if the band is stretched? (b) What happens to the length if the temperature of the band is raised while the tension is constant?

Explain in as much detail as you can how the behaviour can be understood in terms of the microscopic structure of rubber. Compare the mechanical properties of rubber with those of an ideal gas.

4 In a system of non-interacting fermions, the probability that a state is occupied is given by the function $p_f(\varepsilon) = 1 / [\, exp\{(\varepsilon - \mu(T)\,) / kT\} + 1]$.

For two energies ε_1 and ε_2, where $\varepsilon_2 - \mu = \mu - \varepsilon_1$, show that $p_f(\varepsilon_1) = 1 - p_f(\varepsilon_2)$. Use the result to demonstrate that the dependence of μ on T can be ignored, if the density of states $g(\varepsilon)$ is sufficiently slowly varying over an energy range of order kT on either side of $\varepsilon_f = \mu(0)$.

Confirm that this is likely still to be a good assumption even at room temperature for the valence electrons in a typical metal where ε_f = 5eV.

Using the above results, show that the temperature dependence of the internal energy U is described by

$$U(T) = U(0) + 2\, g(\varepsilon_f)\, (kT)^2 \int_0^\infty x\, dx / (e^x + 1)$$

5 Show that the mean energy of an electron in a metal (at T=0K) is $\overline{E} = 3E_F/5$. Explain why the Fermi level increases when a metal is compressed. Show the Bulk modulus of conduction electrons, with density n, is

$$B = -V(\partial P/\partial V)_T = 2nE_F/3$$

Calculate the value for copper and compare the result with: (a) the experimental value of 1.4 x 10^{11} N m^{-2}.; (b) the bulk modulus of a perfect gas at STP (N.B. in an adiabatic change dU = -pdV. For Cu, atomic weight = 63.5; density = 8900 Kg m^{-3}.)

6 For potassium, the Fermi energy is 2.1eV. Estimate the paramagnetic contribution to the magnetic susceptibility of potassium.

7 A typical star has a mass of 3 x 10^{30} Kg, a radius of 3 x 10^7m and a temperature of 10^7 K. The star is composed mainly of dissociated hydrogen atoms so that the

electrons and protons each form a Fermi gas. Estimate the Fermi energy and show the Fermi gases degenerate (i.e $T<T_F$). (Check that it is correct to treat the particles as non-relativistic.)

8 Give an account of the propagation of atomic vibrations along a monatomic chain of atoms, mass m, in which the spacing between atoms is a and in which atoms, distance na from each other are connected by a force constant k_n.

Show that phonons in a 1-D chain with nearest, and next-nearest, harmonic interactions have frequencies given by

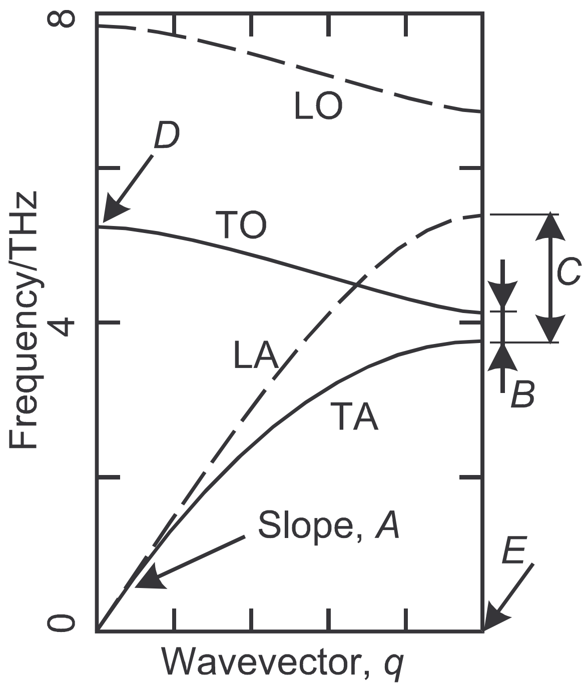
$$\omega^2 = \frac{4}{m}\left(k_1 \sin^2(qa/2) + k_2 \sin^2(qa)\right).$$

Longitudinal waves in Lead are observed to have a maximum phonon frequency at $qa = 0.8\,\pi$, where a is the appropriate interplanar spacing. Estimate the ration of force constants k_1/k_2 and comment on the result.

9 Explain in qualitative terms what is meant by the optical and acoustic branches of a phonon spectrum. Using arrows to represent atomic displacements in magnitude and direction, indicate the relative motion of atoms corresponding to longitudinal phonon modes near the centre and at the boundary of the Brillouin zone for a 1-D solid having two atoms (mass $m_1 > m_2$) per primitive cell.

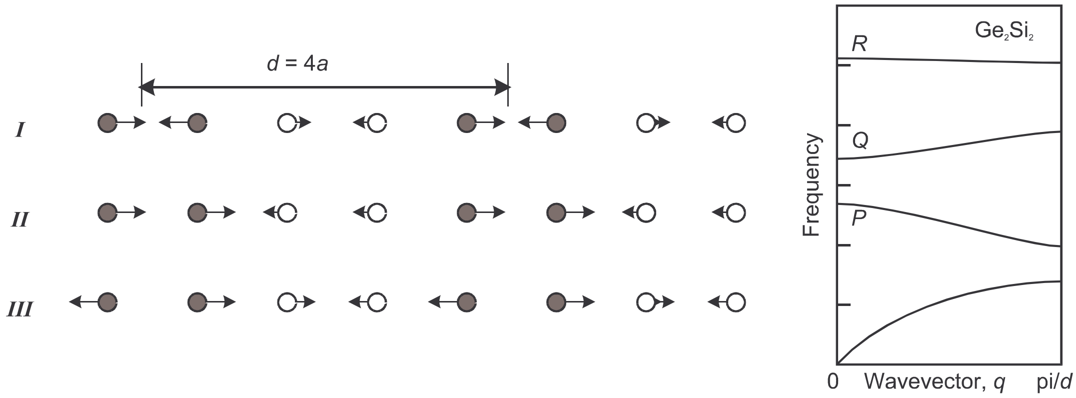
The figure shows the phonon dispersion curves for sodium chloride, NaCl, in the <111> direction. Explain how the parameters A to E are related to the properties of the material and explain how they might be measured.

Discuss briefly the extent to which the diagram could be used to calculate: (i), the specific heat, and, (ii), the thermal conductivity.

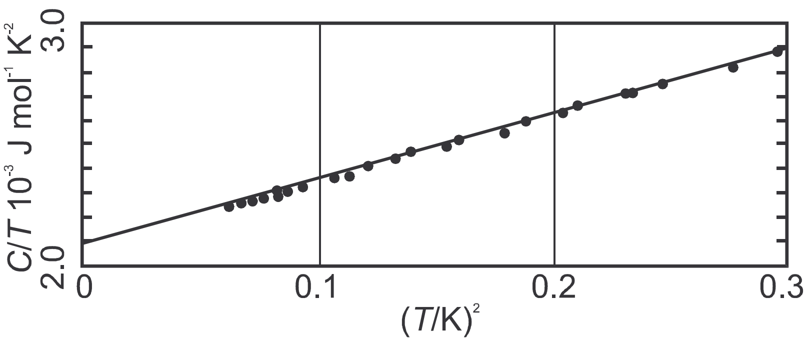


10* A silicon-germanium superlattice, Si_2Ge_2, has a layered structure with two layers of silicon atoms alternating with two layers of germanium. Phonons with q perpendicular to the atomic planes can be treated using a 1-D, harmonic-force model. The four phonon modes that result have dispersion curves show in the diagram below (right). Starting from the dispersion curve for a monatomic chain, indicate, qualitatively, how the 4 modes arise from zone-folding (The argument is an extension of that given in lectures to explain the origin of optical modes in a diatomic lattice).
Displacement patterns for the three modes at $q = 0$, labelled P, Q, R, are sketched in

the left of the diagram, where they are labelled *I*, *II*, *III*, not necessarily in the same order. How do P, Q, and R correspond to *I*, *II*, and *III*?



11 What are the two main contributions to the thermal capacity of a metal, and what is the temperature dependence of each contribution in the low-temperature limit? Explain why the low-temperature thermal capacity for a metal is often plotted in the form C/T against T^2. The graph below shows such data for potassium below approximately 0.5K. Deduce the density of states at the Fermi level, $g(E_F)$, and the Debye Temerautre, θ_D for potassium. Compare these values respectively with the value of $g(E_F)$ calculated in the free electron approximation and with the accepted values of θ_D = 100K. (For K, atomic weight = 39, density = 860 Kg m^{-3})



12 Give an explanation of the terms *order parameter* and *mean-field theory* in the context of phase transitions. Illustrate your answer with two examples of phase transitions that may be described in a mean field approximation.

In one mean-field model of a magnetic phase transition in a crystalline solid, the order parameter, s, is the average magnetisation per site. It can take values between -1 and +1. At temperature, T, the free energy per site is given by

$$F = \frac{zJ}{2}(1+s)(1-s) - \frac{kT}{2}\left[2\ln 2 - (1+s)\ln(1+s) - (1-s)\ln(1-s)\right],$$

where J is an interaction energy and z is the number of nearest neighbours of each site. Explain, without giving detailed derivations, the origin of the two terms in F and show that one favours values of s near zero while the other favours extreme values of s.

Use sketches of the free energy to show the nature of the phase transition that is predicted. Sketch the temperature dependence of the order parameter.

Show that the order parameter is given by the solutions of $\frac{1}{kT} = \frac{1}{2zJs}\ln\left(\frac{1+s}{1-s}\right)$.

Demonstrate that the critical temperature, T_c, is given by $T_c = zJ/k$.
(Based on IB Adv. Physics, 1999, Paper 3, B10)

13 A simple non-linear, dynamical system is described by the differential equation

$$\dot{x} = rx - x^3,$$

where x is the dynamical variable and r is a control parameter that determines the behaviour. Sketch $\dot{x}$ vs. x for $r<0$ and $r>0$ and identify the equilibrium solutions ($\dot{x} = 0$). Distinguish between stable and unstable solutions. Sketch the values of x, at equilibrium, as a function of r. What has this to do with phase transitions?