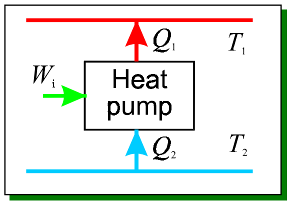
Heat pumps(refrigerators)

(Handout section 4.3)

- An engine in reverse
  - Work is input and the effect is to transfer heat from the cool reservoir to the hot reservoir.



  - For an ideal heat pump:

$$W_i = Q_1 - Q_2 = Q_1\left(1-\frac{Q_2}{Q_1}\right)$$

$$W_i = Q_1\left(1-\frac{T_2}{T_1}\right)$$

e.g. Power to supply 1kW to a house at 293K from a pond at 275K; P=(1-275/293)kW=62W.

  - As a refrigerator:

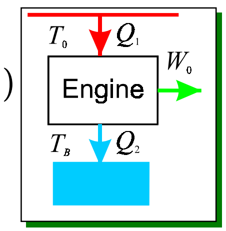
$$W_i = Q_2\left(\frac{Q_1}{Q_2}-1\right)=Q_2\left(\frac{T_1}{T_2}-1\right)$$

e.g. A heat loss of 10W in liquid He dewar, at 4K, requires power (293/4-1)10=720W.

Applications of Carnot cycle

(Handout section 4.4)

- Applications extend well beyond heat engines. Some examples;
- Minimum work to liquefy a gas.
  - Consider the reverse process. i.e. how much work can we extract from a liquid at its boiling point, T_B, equilibrating at room temperature.
  - Two stages (both use a Carnot engine):



    - Vapourise at T_B.

$$Q_2 = L, \quad Q_1 = Q_2(T_0/T_B)$$

$$W_o^{(1)} = L(T_o/T_B - 1)$$

    - Heat to room temp, T_0, at constant pressure.

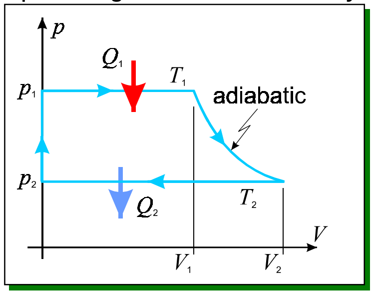
$$W_o^{(2)} = \int_{T_B}^{T_o} c_p(T_o/T - 1)\,\mathrm{d}T$$

$$= c_p T_o \ln(T_o/T_B) - c_p(T_o - T_B)$$

    - Total work $W=W_o^{(1)}+W_o^{(2)}$ is also the minimum to liquefy the gas.
    - For N_2 (take c_p=7R/2, L=5.5 kJ/mol, T_B=77K). W = 21 kJ/mol = 0.17 KWh/litre.

Applications cont…….

- **Stefan's Law**
  - Use photon gas in a reversible cycle:



  - We can calculate the WD in one cycle, using $pV^{-4/3}=\mathrm{const}$ for the adaibatic section.
  - Heat input from hot reservoir is the work-done to expand the photon gas + the change in internal energy of the photon gas.
  - As an engine it must have the same efficiency as a Carnot engine.
  - Leads directly and simply to:

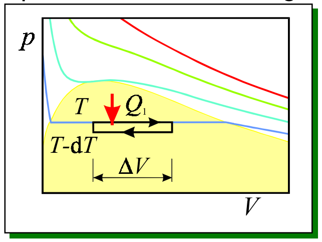
$$u(T)\propto T^4$$

Stefan's Law

originally discovered experimentally

Applications cont….

- **Clausius-Clapeyron Equation: phase co-existence.**
  - Take a Carnot engine round the cycle indicated (in the phase co-existence region).



  - The efficiency is that of a Carnot engine:

$$\eta = 1-\frac{T_1}{T_2} = 1-\frac{(T-\mathrm{d}T)}{T} = \frac{\mathrm{d}T}{T}$$

  - We also have

$$\eta = \frac{W_o}{Q_1} = \frac{dp\Delta V}{L(T)}$$

Area of loop

Latent heat

  - Re-arranging gives, along the phase boundary,

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L(T)}{T\Delta V}$$